An Absolute-Position-Aided Code Discriminator Towards GNSS Receivers for Multipath Mitigation

Yiran Luo, *The Hong Kong Polytechnic University*
Li-Ta Hsu, *The Hong Kong Polytechnic University*
Yan Xiang, *Shanghai Jiao Tong University*
Bing Xu, *The Hong Kong Polytechnic University*
Chunyang Yu, *The University of Calgary*

BIOGRAPHY (IES)

Yiran Luo received the B.S. degree in Electronic Information Science and Technology from Sichuan University, China, in 2014, and the joint Ph.D. degree in Geomatics Engineering from The University of Calgary, Canada, and in Information and Communication Engineering from Beijing Institute of Technology, China, in 2020. She is currently a post-doctoral researcher with the Department of Aeronautical and Aviation Engineering, The Hong Kong Polytechnic University. Her research interests include GNSS signal processing, GNSS software-defined radio (SDR) design, and navigation and localization in harsh environments.

Li-Ta Hsu received the B.S. and Ph.D. degrees in aeronautics and astronautics from National Cheng Kung University, Taiwan, in 2007 and 2013, respectively. He is currently an assistant professor with the Department of Aeronautical and Aviation Engineering, Hong Kong Polytechnic University, before he served as post-doctoral researcher in Institute of Industrial Science at University of Tokyo, Japan. In 2012, he was a visiting scholar in University College London, U.K. He is an Associate Fellow of RIN. His research interests include GNSS positioning in challenging environments and localization for pedestrian, autonomous driving vehicle and unmanned aerial vehicle.

Yan Xiang is an assistant professor at Shanghai Jiao Tong University. She obtained her Ph.D. in the Department of Geomatics Engineering at University of Calgary. Her current research interests include inter-frequency and inter-system bias determination, carrier-phase based ionospheric modeling, and ionospheric augmentation for high precision positioning.

Bing Xu is a Research Assistant Professor with the Department of Aeronautical and Aviation Engineering (AAE), The Hong Kong Polytechnic University. He was a Postdoctoral Fellow with the same institution from August 2018 to April 2020. His current research is the signal processing with applications to Positioning and Wireless Communications.

Chunyang Yu received PhD degrees in Positioning, Navigation and Wireless Location from the University of Calgary (Calgary, Canada). Dr. Yu is currently a Postdoc Fellow in the Mobile Multi-Sensor Systems (MMSS) research group at the University of Calgary, and her research is focused on inertial sensor based navigation system, specifically pedestrian navigation system, autonomous vehicle navigation systems.

ABSTRACT

The multipath effect can severely reduce the positioning accuracy of GNSS receivers. Thus, it is very important to estimate and mitigate the multipath delay error in tracking algorithms, especially in harsh environments. The code phase error is highly related to the positioning performance. Especially, the absolute position results are dependent on the absolute term of the code phase. However, the absolute code phase is seldom discussed and processed in state-of-the-art methods. In this paper, the absolute position is proposed to form a code phase discriminator to estimate the missing absolute code phase. In this discriminator, the standard vector tracking delay/frequency lock loop (VDFLL) is also used for compensating the change of the code phase error. Besides, the multipath error is estimated with the between-satellite single difference algorithm to further improve the proposed absolute-position-aided (APA) VDFLL code phase discriminator. The real-world static field test demonstrates the proposed algorithm's high performance in positioning and multipath mitigation.

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INTRODUCTION

Multipath delay errors frequently happen to the global navigation satellite (GNSS) receivers for current state-of-the-art applications in navigation. As the GNSS signals are confronted with the weak receiving power and the code and carrier auto-correlation interferences induced by the line-of-sight (LOS) and non-line-of-sight (NLOS) paths [1], the multipath effect can cause one of the most severe issues to the design of receivers. The way to mitigate such errors inevitably plays a very important role for the design of the next-generation GNSS receiver.

There have been many algorithms presented to strengthen the receiver's ability towards multipath mitigation in the baseband design. For example, the multi-correlators are used to predict the curves of the discriminated errors by which the multipath signal can be estimated [2]. The use of antenna array is also very efficient to deal with the multipath problem as the LOS and NLOS can be identified by the direction-of-arrival (DOA) algorithms in this way [3]. However, these two methods will highly increase the hardware cost of receivers. Direct position estimation (DPE) takes advantages of the spatial information of the GNSS satellites by optimizing the discriminating error with the user's 3D position and it has been proved to be superior to the traditional algorithms for the multipath mitigation [4]. It attracts lots of attentions of the researchers in recent times for its state-of-the-art concept applied to the GNSS receiver design [5]. Vector tracking algorithms are also very popular to deal with the interfered signals in the receiver design [6] and it has been recently proved to be efficiently classify the LOS and NLOS signals [7]. Nevertheless, both of the DPE and the vector tracking algorithms fail to deal with the absolute code phase error which can still reduce the position accuracy especially in a harsh environment.

The paper focuses on dealing with the absolute code phase error and improving this error estimation in the discriminator by modeling the multipath error. This algorithm requires less computational load compared with the DPE and less hard-ward cost than the antenna-array-based algorithms. In comparison with the standard vector delay/frequency lock loop (VDFLL) algorithm [8], there is little extra computational load adding to the baseband design. Thus, it is a promising method to upgrade the next-generation GNSS receiver. The proposed APA VDFLL algorithm even has the potential to efficiently improve the receivers for the signals of opportunity (SOOP) in the future.

Therefore, the main contributions of this paper can be summarized as follows:

1. An absolute code phase discriminator is proposed using the absolute position information and VDFLL;
2. The between-satellite single difference algorithm is proposed to model the multipath error such that the proposed APA VDFLL discriminator can be further improved;
3. The real-world static experiments are implemented to verify the performance of the multipath mitigation at both tracking and positioning stages.

METHODOLOGY

In this section, the code discriminator based on the absolute position and the VDFLL algorithm is firstly presented. Then, the estimator for multipath delay error in the absolute code phase error will be investigated. At first, the architecture of the standard VDFLL GNSS receiver is shown in Figure 1 where the notations are listed in appendix. The blue blocks and lines relate to the aiding process with the relative positioning information. In this approach, both carrier and code Doppler frequencies are well compensated by the predicted user's velocity. Then, the architecture of the GNSS receiver based on the proposed APA VDFLL approach is illustrated in Figure 2 and it is an upgraded version of the standard VDFLL receiver. In the figure, the red blocks and lines represent the absolute positioning aiding while the blue ones denote the algorithm of the relative positioning aiding mentioned in Figure 1. In this navigation system, the tracking loop is assisted with both the relative position (user's velocity) and the absolute position. Furthermore, the multipath mitigation algorithm is presented based on the APA algorithm blocks in this work as well. Detailed algorithms will be introduced in the subsequent part. It is worth mentioning that the user's velocity and position can be estimated by the non-linear least square (NLS) method in a standalone GNSS receiver [9]. In this study, a ground truth will be provided as the absolute position feedback in order to efficiently verify the proposed algorithm.

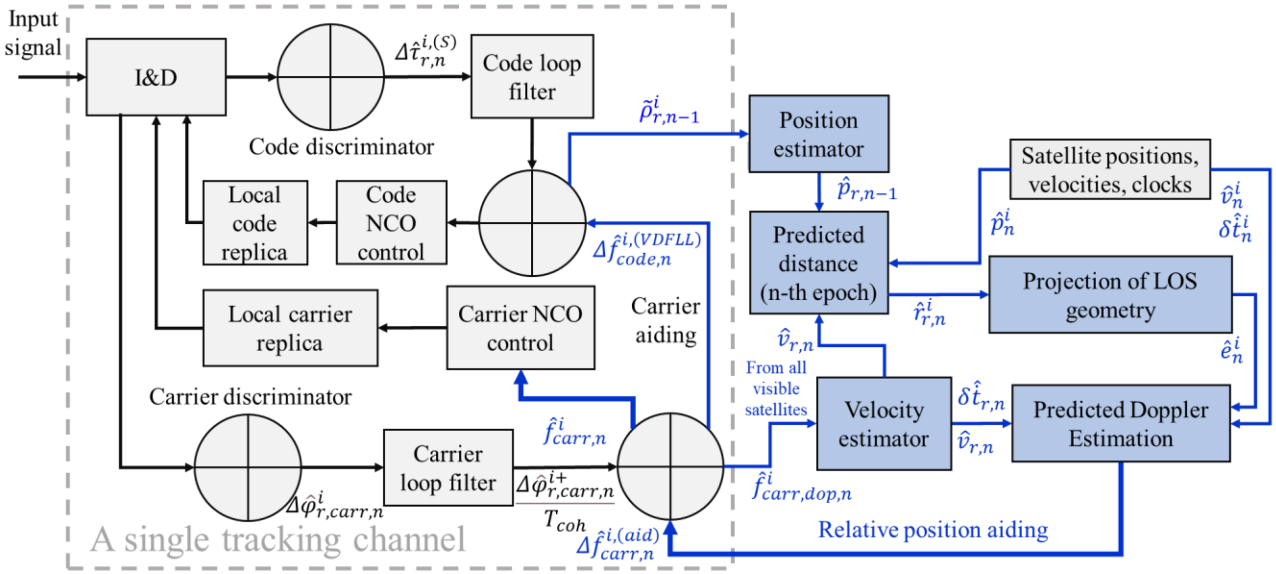

Figure 1 Architecture of the GNSS receiver based on the standard VDFLL algorithm.

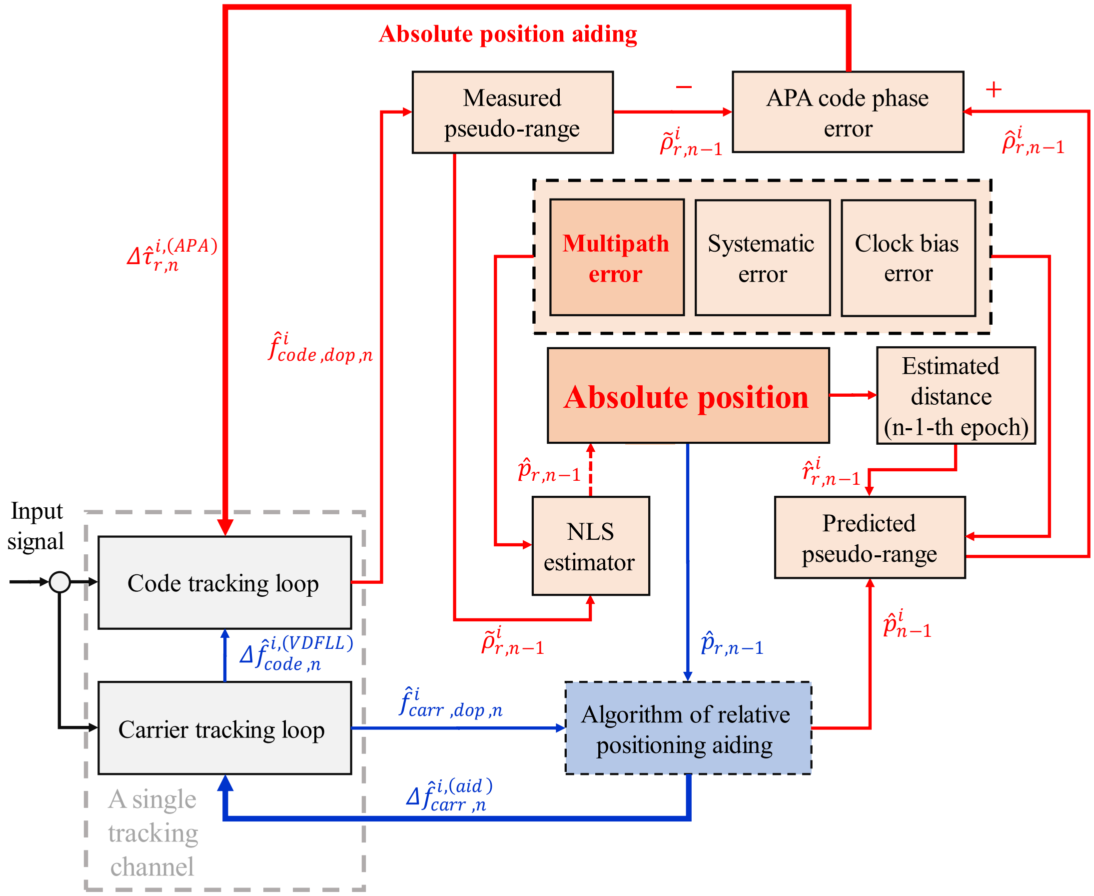

Figure 2 Architecture of the GNSS receiver based on the proposed APA VDFLL algorithm for multipath mitigation.

Absolute-Position-Aided Code Discriminator

The proposed code discriminator aided with the absolute position is given by

$$\Delta\hat{\tau}_{r,n}^{i} = \Delta\hat{\tau}_{r,n}^{i,(S)} + \Delta\hat{\tau}_{r,n}^{i,(APA)} \tag{1}$$

with

$$\Delta\hat{\tau}_{r,n}^{i,(APA)} = \frac{f_c}{c}\left(\tilde{\rho}_{r,n-1}^{i} - \hat{\rho}_{r,n-1}^{i}\right) \tag{2}$$

where

superscript i corresponds to the pseudo-random noise (PRN) number of the satellite for the tracking channel;

$\Delta\hat{\tau}_{r,n}^{i,(S)}$ is the output of the noncoherent early-minus-late-envelope code discriminator [1];

$\Delta\hat{\tau}_{r,n}^{i,(APA)}$ is the APA code phase error prediction in chips at the n-th epoch;

f_c and c are the code frequency and the speed of light, respectively;

$\hat{\rho}_{r,n-1}^{i}$ and $\tilde{\rho}_{r,n-1}^{i}$ are the estimated and measured pseudo-ranges at the $n-1$-th epoch, respectively where the latter is dependent on the incoming GNSS signal while the former is formed with the geometry distance based on the absolute position and error source models of the pseudo-range measurements.

Based on the proposed estimator (2), it can be found that the absolute position information is exploited by the model of the pseudo-range estimation $\hat{\rho}_{r,n-1}^{i}$ such that the discriminator (1) can be improved in this way.

In this paper, the pseudo-range measurement $\tilde{\rho}_{r,n-1}^{i}$ which is updated and produced based on the code numerically controlled oscillator (NCO) algorithm is aided with the VDFLL algorithm in the GNSS software-defined receiver (SDR)**.** Then, the code frequency for the NCO algorithm is given by

$$\hat{f}_{code,n}^{i} = f_c + \hat{f}_{code,dop,n}^{i}$$

with

$$\hat{f}_{code,dop,n}^{i} = \Delta\hat{f}_{code,n}^{i,(VDFLL)} + \frac{1}{T_{coh}}\Delta\hat{\tau}_{r,n}^{i+} \tag{3}$$

$$\Delta\hat{f}_{code,n}^{i,(VDFLL)} = -\frac{f_c}{f_r}\hat{f}_{carr,dop,n}^{i}$$

$$\hat{f}_{carr,dop,n}^{i} = \Delta\hat{f}_{carr,n}^{i,(aid)} + \frac{1}{T_{coh}}\Delta\hat{\varphi}_{r,carr,n}^{i+}$$

where

$\hat{f}_{code,dop,n}^{i}$ is the code Doppler frequency;

T_{coh} is the coherent integration time or the updating interval of the tracking in this work;

$\Delta\hat{\tau}_{r,n}^{i+}$ is the output of a 2-nd order loop filter [1] of which the input is $\Delta\hat{\tau}_{r,n}^{i}$ as given by (1);

$\Delta\hat{f}_{code,n}^{i,(VDFLL)}$ is the carrier aiding for the code Doppler estimation based on the VDFLL algorithm;

f_r is the radio frequency of the incoming GNSS signal;

$\hat{f}_{carr,dop,n}^{i}$ is the carrier Doppler estimation;

$\Delta\hat{f}_{carr,n}^{i,(aid)}$ is the aiding of the carrier Doppler computed by the user's 3D velocity prediction at n-th epoch and its corresponding velocity of the LOS satellite i;

$\Delta\hat{\varphi}_{r,carr,n}^{i+}$ is the carrier phase error filtered by a 1-st order loop and its input is given by a Costa loop discriminator [1].

As explained earlier, the VDFLL is responsible for compensating the change of the code phase error in tracking while the APA algorithm can properly deal with the absolute code phase error. More details about the VDFLL algorithm can refer to the authors' previous works [9], [10].

Estimator of the Multipath Delay Error

In this work, the estimated pseudo-range in (2) is given by

$$\hat{\rho}^{i}_{r,n-1} = \hat{r}^{i}_{r,n-1} + c \cdot \left(\delta\hat{t}_{r,n-1} - \delta\hat{t}^{i}_{n-1}\right) + \hat{B}^{i}_{a,n-1} - \kappa_D \cdot \hat{B}_{r,mp,n-1} \tag{4}$$

with

$$\hat{r}^{i}_{r,n-1} = \left\| \hat{\mathbf{p}}^{i}_{n-1} - \hat{\mathbf{p}}_{r,n-1} \right\|$$

where

$\hat{r}^{i}_{r,n-1}$ is the prediction of the geometry distance between the satellite i and the user r;

$\hat{\mathbf{p}}^{i}_{n-1}$ is the 3D position of satellite i which is computed by the ephemeris;

$\hat{\mathbf{p}}_{r,n-1}$ is the estimated user's absolute 3D position; it is obtained from a ground truth for a clear verification;

$\delta\hat{t}_{r,n-1}$ and $\delta\hat{t}^{i}_{n-1}$ are the estimated local and satellite clock bias error in seconds, respectively;

$\hat{B}^{i}_{a,n-1}$ is the estimated atmospheric delay error in meters;

$\hat{B}_{r,mp,n-1}$ denotes the estimation of the multipath-induced biased error here in meters and κ_D is a coefficient which is varied with the design of the code phase error discriminating algorithm and it is decided by the early-late spacing here.

Therefore, the between-satellite single difference algorithm is proposed to obtain the multipath delay error as

$$\hat{B}^{i}_{r,\rho,mp,n-1} = \left(\tilde{\rho}^{i}_{r,n-1} - \hat{r}^{i}_{r,n-1} - c \cdot \left(\delta\hat{t}_{r,n-1} - \delta\hat{t}^{i}_{n-1}\right) - \hat{B}^{i}_{a,n-1}\right) - \left(\tilde{\rho}^{m}_{r,n-1} - \hat{r}^{m}_{r,n-1} - c \cdot \left(\delta\hat{t}_{r,n-1} - \delta\hat{t}^{m}_{n-1}\right) - \hat{B}^{m}_{a,n-1}\right) \tag{5}$$

where $\tilde{\rho}^{i}_{r,n-1}$ corresponds to the LOS satellite i which is contaminated by the multipath interference while $\tilde{\rho}^{m}_{r,n-1}$ corresponds to the master LOS satellite m which is assumed to be not affected by the multipath effect; other remained estimations with superscript m are related to the master satellite at the $n-1$-th epoch.

In summary, the code frequency estimation is improved with (3) aided by the APA VDFLL discriminator. Furthermore, the multipath delay error is able to be mitigated in the proposed algorithm with (5). It is also worth noting that once the absolute position estimation as well as the other error source models are accurate enough, the multipath error can be almost eliminated based on (4).

FIELD TEST EXPERIMENT

The experimental equipment is set up as shown in Figure 3 where the test situation is stationary. A NovAtel antenna is used to receive the GNSS signals through a GNSS receiver front-end. A U-Blox M8T receiver shares the same antenna for the front-end. The ground truth position of this test is provided by the real-time kinematic (RTK) positioning algorithm by the U-Blox receiver where the baseline is assumed to be short enough for the RTK algorithm in this experiment. Also, the estimations of $\delta\hat{t}_{r,n-1}$, $\delta\hat{t}^{i}_{n-1}$ and $\hat{B}^{i}_{a,n-1}$ are obtained based on the base station and they are assumed to be accurate enough. It should be mentioned that the absolute position used for the APA VDFLL algorithm in this work is the true position just for well verifying the performance of the proposed GNSS SDR. The parameter settings for the GNSS SDR used in this work is listed in Table 1. It is worth mentioning that, based on a post processing, it can be known in advance that only PRN 10, PRN 20, and PRN 27 channels are affected by the multipath effect in a more severe way in this experiment, so only these three channels are improved by the proposed APA method when the algorithm is activated in a GNSS receiver for a clear assessment (the VDFLL algorithm is applied to all the channels). A stationary scenario is determined for evaluating the proposed APA VDFLL algorithm here. The test spot and the sky plot are shown in Figure 4. The satellite of PRN 21 is selected as the master satellite to estimate the multipath error proposed by (5).

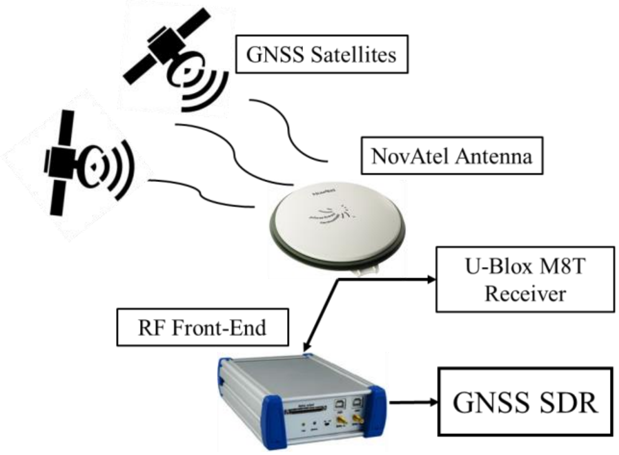

Figure 3 Set-up of the static field test.

Table 1 Parameter settings for the static field test.

Terms	**Values**
Signal type	GPS L1 C/A
IF sampling rate	10.125 MHz
Radio frequency	1575.42 MHz
Code frequency	1.023 MHz
Quantization type	int8, complex
Front-end oscillator	TCXO
Coherent integration time	5 ms
Bandwidth of the code loop filter	0.4 Hz
Bandwidth of the carrier loop filter	4 Hz
Early-late code discriminating spacing over one code cycle	0.4 chips
Master satellite	PRN 21
Channels based on the APA method for multipath mitigation (when the proposed APA VDFLL algorithm is used in the GNSS SDR)	PRN 10, PRN 20, PRN27

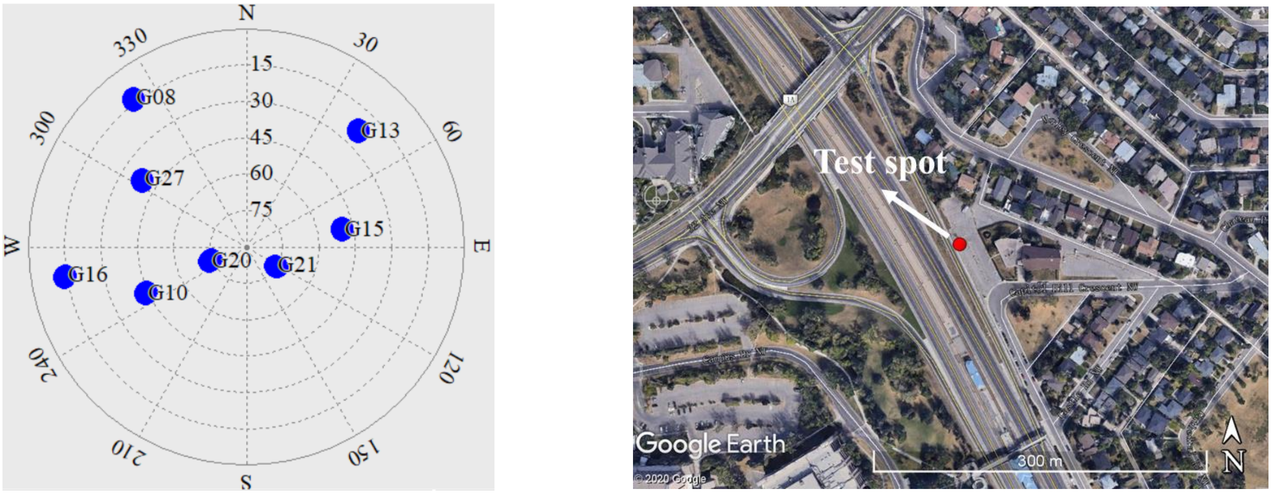

Figure 4 Sky plot of the LOS satellites (left) and the test spot (right).

Tracking Results

The tracking results related to the code phase error are evaluated with different GNSS SDR algorithms in this part. The tracking channels for PRN 10, PRN 20, and PRN 27. The probability density function (PDF) curves of the discriminated and filtered code phase errors, i.e., $\Delta\hat{\tau}_{r,n}^{i}$ and $\Delta\hat{\tau}_{r,n}^{i+}$, are plot in Figure 5. As can be found from the scalar tracking results, these three channels are severely affected by the multipath effect on the code signal as the shapes of the PDF curves are combined with more than one white Gaussian distributions (approximated from the results). On the one hand, according to the tracking results, as the expectation of the standard VDFLL becomes closer to the expectation of the biased error in the scalar tracking results, it can be proved that this algorithm has the ability to mitigate the multipath effect. However, some remained code phase errors at the same epoch which is not able to be compensated by the carrier aiding algorithm still reduces the tracking accuracy. Then, the proposed APA VDFLL can properly estimate the remained code phase error in the discriminating process, as the expectation of the discriminating errors approaches the ones of the biased errors more than the results produced by the standard VDFLL. Similarly, the unexpected remained multipath error is further estimated by (5) in this work and removed from the proposed APA code phase discriminator (2) by (4). The experimented tracking results have also demonstrated the high performances of the proposed multipath error estimator when they are compared with the APA-based results without this estimating process.
Next, as illustrated in Figure 6, it can be found that the C/N_0 estimating results perform different when different tracking algorithms are used. The mean of the C/N_0 estimations are also listed in Table 2. First of all, all the estimations using the scalar tracking have the highest value among the four algorithms. When the multipath interference occurs to an incoming signal, the power peak of the summation sample does not correspond to the real code phase error, as the standard incoming signal model is totally destroyed by the extra NLOS signal source. In this case, the real C/N_0 value should be averagely lower than the productions from the scalar GNSS receiver. It is obvious that PRN 10 and PRN 27 embrace the lowest C/N_0 estimations from the APA VDFLL algorithms when compared with the scalar tracking and the VDFLL algorithm. Furthermore, when the multipath error is estimated, the performance of the estimating results can be further improved. For PRN 20, among the three tested satellites, it has the highest elevation angle but its C/N_0 estimations are the lowest. It can be roughly inferred that the multipath affect has occurred but the LOS signal of PRN 20 is just slightly affected by the incoming NLOS signal. Again, from the mean results listed in Table 2, it can be seen that the tracking performance is slightly improved by the proposed APA VDFLL algorithms as well. It can be proved that the proposed algorithms are sensitive to process both strong and weak multipath interference cases.

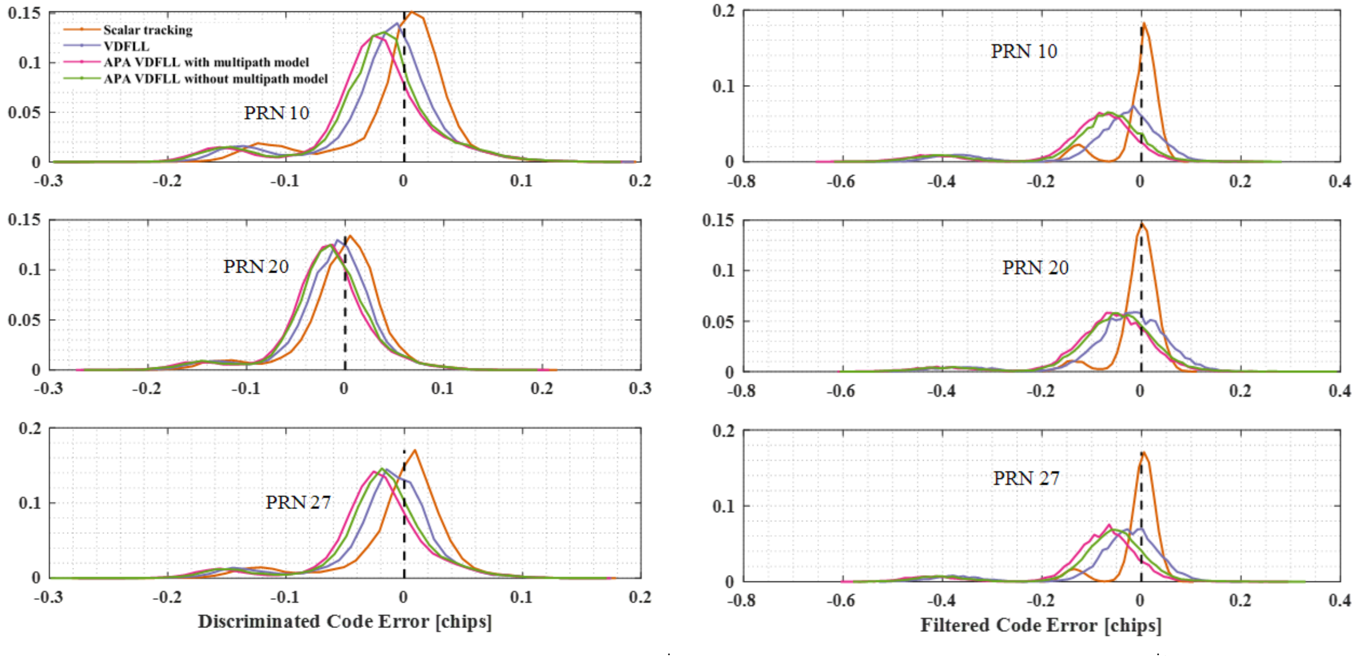

Figure 5 PDF curves of the discriminated code phase error $\Delta\hat{\tau}_{r,n}^{i}$ (left) and the filtered code phase errors $\Delta\hat{\tau}_{r,n}^{i+}$.(right)

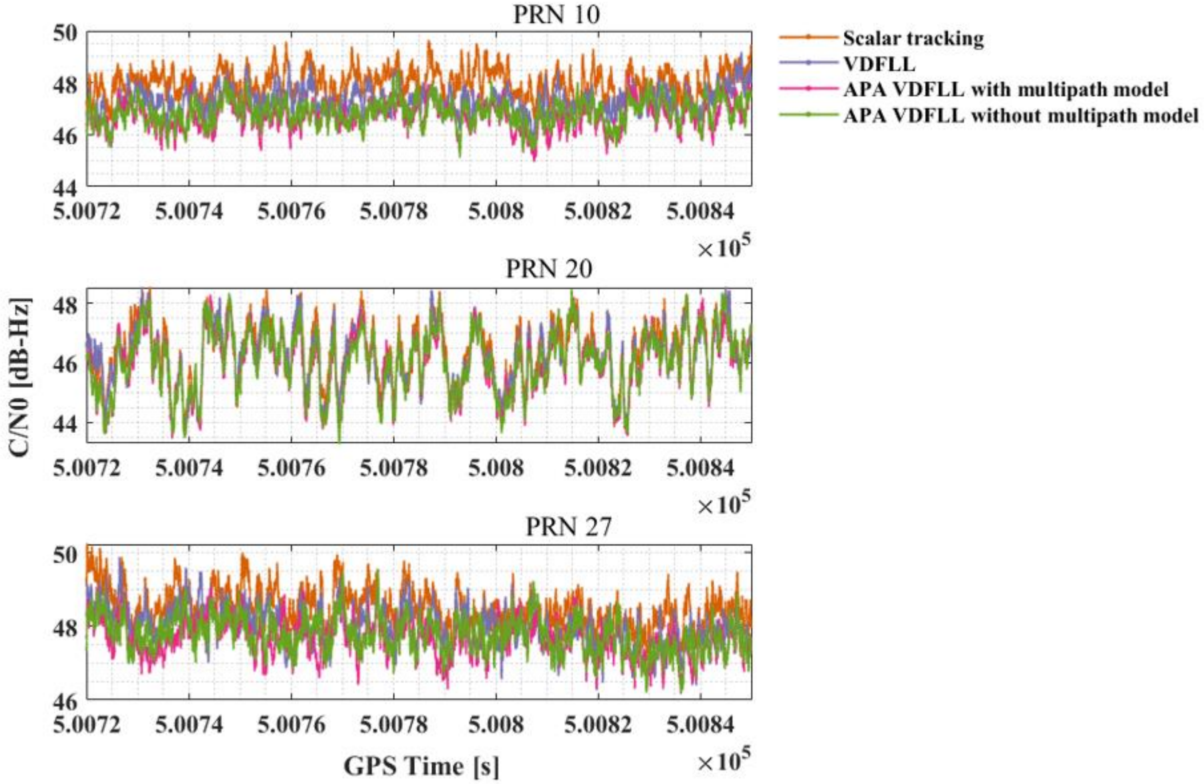

Figure 6 $\mathrm{C/N_0}$ estimation results with different GNSS SDR algorithms.

Table 2 Means of $\mathrm{C/N_0}$ estimations for different GNSS SDR algorithms.

Satellite PRN number	**Scalar tracking**	**VDFLL**	**APA VDFLL without multipath estimation**	**APA VDFLL with multipath estimation**
10	48.08 dB-Hz	47.23 dB-Hz	46.85 dB-Hz	46.7 dB-Hz
20	46.45 dB-Hz	46.19 dB-Hz	46.03 dB-Hz	45.98 dB-Hz
27	48.43 dB-Hz	48.04 dB-Hz	47.84 dB-Hz	47.73 dB-Hz

DGNSS Positioning Results

In order to test the performance of the proposed APA VDFLL algorithms on the navigation solutions, the differential GNSS (DGNSS) positioning algorithms are also tested for the different GNSS SDRs in this work. The open-source package, RTKLIB v2.4.3 b31, is used to implement the DGNSS algorithm [11].

Firstly, the DGNSS positioning results in the horizontal plane and in the local level frame are shown in Figure 7 and Figure 8, respectively. The scalar tracking results perform the worst and the positioning results are efficiently improved by the VDFLL algorithm. Furthermore, when the APA VDFLL algorithm with the multipath error compensation is used, the mean of the DGNSS positioning has the smallest value while the one without compensation is slightly worse than the former. It can be found that both of the two APA VDFLL algorithms are superior to the other two algorithms in term of the positioning accuracy. The results verify that the proposed algorithm can further improve the positioning algorithm and its performance can even be better once the multipath error is able to be removed from the proposed APA discriminator. The proposed APA VDFLL discriminator is accordingly proved to estimate and remove the biased error in the pseudo-range measurement with little computational load increase.

It can be found that, even the proposed APA algorithm is used, there are still obvious biased errors existing in the east direction. One explanation is that the satellites of PRN 10, PRN 16, and PRN 27 which are affected by the multipath effect will have higher influence in the east direction than the north direction according to the geometry distribution shown in Figure 4. Thus, the remained multipath delay error reduces the eastern positioning results more than the northern ones. By optimizing the APA VDFLL discriminator, it can be further improved.

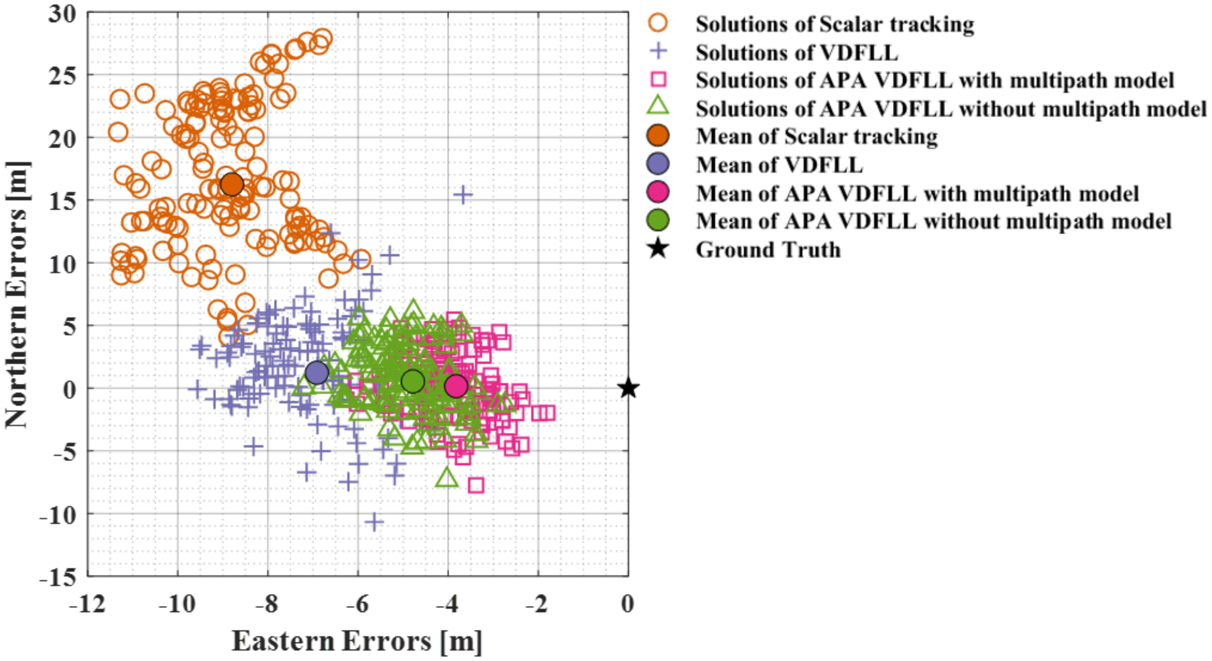

Figure 7 DGNSS positioning results in the horizontal plane.

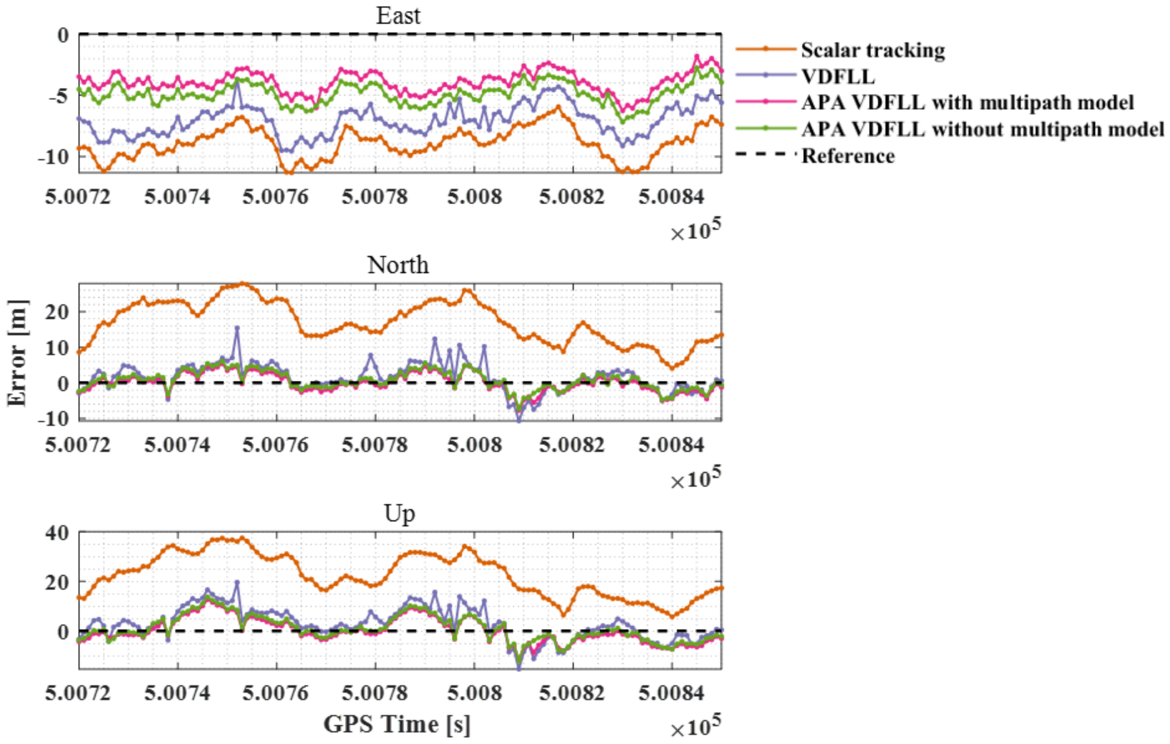

Figure 8 DGNSS positioning results in the local level frame.

CONCLUSIONS

In this paper, an absolute-position-aided discriminator based on vector tracking has been proposed to improve the performances in positioning and multipath mitigation of the GNSS receiver. In the proposed discriminator, the APA algorithm is used for the compensation of the absolute code phase error while the VDFLL accounts for change of such error. Then, the between-satellite single difference algorithm has been introduced in the proposed APA discriminator for further eliminating the remained multipath delay error. The static field test has been implemented to verify the APA VDFLL approach. Compared with the traditional algorithms, both of the tracking and positioning experimental results demonstrate that the proposed APA VDFLL algorithm has the highest performance on positioning accuracy and multipath mitigation.

APPENDIX

Notations

n	Index of the tracking interval
i	PRN number of the satellite
T_{coh}	Coherent integration time of tracking
$\hat{f}^{i}_{code,n}$	Predicted code frequency estimation for the code NCO algorithm
$\Delta\hat{\tau}^{i,(S)}_{r,n}$	Code phase error predicted from a noncoherent early-minus-late-amplitude code discriminator
$\Delta\hat{\tau}^{i,(APA)}_{r,n}$	APA code phase error prediction at the n th epoch (in chips)
$\Delta\hat{\tau}^{i}_{r,n}$	Code phase error predicted from the proposed APA VDFLL discriminator (in chips)
$\Delta\hat{\tau}^{i+}_{r,n}$	Output of a 2-nd order loop filter of which the input is $\Delta\hat{\tau}^{i}_{r,n}$;
$\hat{f}^{i}_{code,dop,n}$	Predicted code Doppler frequency
$\tilde{\rho}^{i}_{r,n-1}$	Measured pseudo-range at $n-1$ th epoch
$\hat{\rho}^{i}_{r,n-1}$	Estimated pseudo-range at $n-1$ th epoch
$\hat{r}^{i}_{r,n-1}$	Estimated geometry distance at the $n-1$ th epoch
$\hat{\mathbf{p}}_{r,n-1}$	Estimated user's absolute 3D position at the $n-1$ th epoch
$\hat{\mathbf{p}}^{i}_{n\text{-}1}$	Estimated satellite's absolute 3D position at the $n-1$ th epoch
$\hat{\mathbf{v}}_{r,n}$	Predicted user's 3D velocity at the n th epoch
$\hat{\mathbf{v}}^{i}_{n}$	Predicted satellite's 3D velocity at the n th epoch
$\delta\hat{\dot{t}}_{r,n}$	Predicted local clock drift error at the n th epoch
$\delta\hat{\dot{t}}^{i}_{n}$	Predicted satellite clock drift error at the n th epoch
$\hat{\mathbf{e}}^{i}_{n}$	Predicted unit direction cosine vector along the LOS direction
$\Delta\hat{f}^{i,(aid)}_{carr,n}$	Aiding carrier Doppler frequency predicted from the vector tracking algorithm
$\Delta\hat{f}^{i,(VDFLL)}_{code,n}$	Aiding code Doppler frequency predicted from the VDFLL algorithm
$\Delta\hat{\varphi}^{i}_{r,carr,n}$	Carrier phase error predicted from the two-quadrant arctangent discriminator (in cycles)
$\Delta\hat{\varphi}^{i+}_{r,carr,n}$	Output of the 1-st order loop filter of which the input is $\Delta\hat{\varphi}^{i}_{r,carr,n}$;
$\hat{f}^{i}_{carr,n}$	Predicted carrier frequency for the carrier NCO algorithm

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